

Collaborative Mathematics Learning

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The collaborative approach to teaching mathematics uses methods that allow the student to participate in building their own knowledge. The shift is from passive assimilation to active engagement. The guide on the side, not the sage on the stage. I will describe here the practical elements I have used in teaching over the past thirty years to achieve this active engagement.

Collaborative Tutorial

Let me describe a typical course, say first year calculus or linear algebra. The course revolves around a weekly tutorial session. This is the most important event of the week and students are expected to attend. And they do attend because they quickly learn that an active tutorial session will give them the tools to master that week's work and is also fun.

At the tutorial, the students work on specially designed progressive problem sets that lead them through the construction of a mathematical idea. They will have had a preparatory hint in class but no answers. The tutorial is where they see each idea bloom for the first time.

The problem sets start with simple concrete examples. These are followed by more involved examples. The students then construct general statements about the ideas they have been exploring. The marks decay rapidly down the sheet (say 5-3-1-1-0) so that students will spend quality time on the foundations.

Students work in groups of up to four. They choose their own study buddies and are allowed to change if things don't work out. Lone rangers are also allowed. The group hands in one set of solutions, written by a different member each week.

Each tutorial is marked out of 10 and all members get the same mark. An honest 50 minutes of work will get you 7 out of 10.

The challenges for the instructor include designing the tutorial and managing the dynamics in each tutorial group. You must discourage a group from a divide and conquer approach. In some case the instructor has to encourage group discussion and work. Always include extra questions for the speedsters to work on. This is part of acknowledging that students have different backgrounds, skills, and learning strategies and cover ground at very different speeds.

Class Time

Start each class with a warm-up question.

In class, the instructor makes sure the lesson of the tutorial is understood, answers any questions, talks about how these new ideas mesh with the rest of the course, talks about how these ideas are used to solve interesting problems.

In class, don't talk for more than about 10 minutes without having the students do something active. This break can be achieved by having them solve some problems or participate in a class exercise. Example: give me a continuous function with domain $[0, 1]$ and range $(0, 1)$ and one with domain $(0, 1)$ and range $[0, 1]$. Discuss.

Then the instructor starts the groundwork for the next tutorial.

It is also very helpful to have two or three general "Request Days". Here the students submit questions and the instructor crafts a lesson out of the answers. This is a great way to find out what is actually stumping the class.

A great teaching skill that fits with this approach is to learn how to answer most questions with a new question that the student can answer and then use as the key to the original answer.

Make the agenda perfectly clear and talk about the reasons this subject is important. Also talk about the learning process that the student is experiencing.

Weekly assignment

Each tutorial starts with handing in the weekly assignment and ends with picking up the questions for the next week. The assignment is a short set of practice problems that allow the student to test their knowledge. They are asked to submit their own answers but many of the tutorial groups meet off-line to do the assignments.

Grading the course

The marks come from three sources: weekly tutorials, weekly assignments, and exams.

A student typically has 10 tutorial and 10 assignment marks each term (each student is allowed to drop or skip 2 of each in a term). There are usually two mandatory individual days of reckoning – a mid-term examination and a final examination.

At the end, the student's final grade is calculated from the term mark and the final exam mark or just the final examination mark if this is higher. Interestingly, the cumulative term mark and the final examination mark are almost always very close.

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